# Extracting the Model Parameters of Non-Ideal Junctions Based on Explicit Analytical Solutions of I-V Characteristics

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*Abstract*--- We present a new method to extract the intrinsic and extrinsic model parameters of semiconductor junctions containing parasitic resistance and shunt conductance. The method, which is based on a previously defined Integral Difference Function *D*, uses the exact explicit analytical solutions of the *I*-*V* characteristics. The presence of Lambert W function terms in the explicit analytical solutions would make parameter extraction by direct numerical fitting cumbersome. However, the resulting *D* is reduced to a purely algebraic equation from whose coefficients the intrinsic and extrinsic model parameters are then readily determined. The procedure is illustrated for four cases of parasitic series resistance - shunt conductance combinations, and applied to synthetic *I*-*V* characteristics to illustrate the computation process.

## 1. Introduction

Modeling semiconductor junctions, bipolar and Schottky, often requires considering the presence of parasitic series resistance and shunt conductances [1]. Extracting the intrinsic junction parameters, junction ideality factor and reverse current, from the experimental current-voltage characteristics is often obscured by the presence of these parasitics. This issue has received considerable attention and several extraction methods have been proposed to the effect. Most of them are approximate procedures that rely essentially on extracting each parameter from restricted regions of the *I*-*V* characteristics where some other parameter is assumed to be negligible. Such approaches work as long as distinct regions are actually present in the characteristics, which is not the case when the junction exhibits considerable parasitic series resistance and shunt conductance.

## 2. Exact analytical solutions

Figure 1 presents a circuit model for a diode which includes a single exponential ideal junction, series parasitic resistance ($R_s$), and two parallel parasitic conductances ($G_{p1}$ and $G_{p2}$).

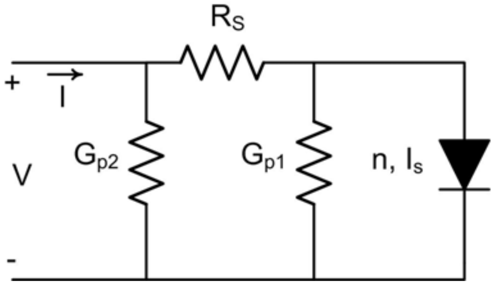

Fig. 1. Generic single-exponential diode equivalent circuit including parasitic series and parallel conductances.

Using this model, the *I*-*V* characteristics are defined by the following equation:

$$I = I_O \left( \exp\left( \frac{V\left(1+R_s\, G_{p2}\right) - I\, R_s}{n V_{th}} \right) - 1 \right) + \left(V - I\, R_s\right) G_{p1} + V\, G_{p2}\left(1 + R_s\, G_{p1}\right) \tag{1}$$

where *I* is the terminal current, *V* is the terminal voltage, $I_O$ is the reverse saturation current, *n* is the ideality factor, and $V_{th} = kT/q$ is the thermal voltage. As it is well known, this transcendental equation may not be solved in general in terms of common elementary functions. Some approximate solutions have been proposed [2, 3] which use elementary functions. However, exact explicit analytical solutions for the current and the voltage already exist [4] in terms of the Lambert *W* function, as presented below:

$$I = \frac{nV_T}{R_S} \mathrm{W}\left\{ \frac{I_0 R_S}{nV_T\left(1+R_S\, G_{P1}\right)} \exp\left[ \frac{\left(V + I_0 R_S\right)}{nV_T\left(1+R_S\, G_{P1}\right)} \right] \right\} + \left( \frac{V\, G_{P1} - I_0}{1 + R_S\, G_{P1}} \right) + V\, G_{P2} \tag{2}$$

and

$$V = -nV_T\, d_2\, \mathrm{W}\left\{ \frac{I_0\, R_{12}}{nV_T\, d_2} \exp\left[ \frac{\left( I + \frac{I_0}{d_2} \right) R_{12}}{nV_T} \right] \right\} + I\, d_2 \left[R_S + R_{12}\right] + I_0\, R_{12} \tag{3}$$

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where $R_{12} \equiv \frac{1}{\left(G_{P1} + G_{P2} + G_{P1} G_{P2}\ R_S\right)}$ has units of resistance, and $d_1 \equiv \frac{1}{\left(1 + R_S\ G_{P1}\right)}$

and $d_2 \equiv \frac{1}{\left(1 + R_S\ G_{P2}\right)}$ are dimensionless.

The above solutions make use of the principal branch of the Lambert *W* function [5], a special function which is not expressible in terms of elementary analytical functions, and is defined as the solution of the equation $W(x)\ \exp[W(x)] = x$. Although the Lambert *W* function is not yet widely used in electronics problems, it has already proved useful in other Physics applications [6,7]. It has also been used for solving some previously analytically unsolved but basic diode [4] and bipolar transistor circuit analysis problems [8], as well as in device modeling formulations [9]. These type of solutions can also be used directly to study illuminated solar cells, as was recently done [10], by simply adding the photogenerated current to the ideal junction current. As can be easily observed, (2) and (3) explicitly represent each variable as a function of the other and the device model parameters. These explicit representations are computationally convenient to use in device models for circuit simulation.

## 3. Parameter extraction

The problem of extracting the model parameters could be attempted by direct vertical optimization of the parameters using the measured *I-V* data by minimizing the quadratic error on the vertical axis (i.e. the current). However this method would be quite computationally intensive because of the implicit nature of the equation. Recently direct lateral optimization was proposed [11] for a diode exhibiting only significant series resistance, based on the approach of minimizing the error on the horizontal axis (i.e., the voltage). The motivation for doing so, in the case of a diode with only series resistance ($G_{p1} \approx G_{p2} \approx 0$), is that the voltage can be explicitly solved from (1) as a function of the current, significantly reducing the computation time.

Other extraction procedures make use of auxiliary functions or operators [12-15]. A useful method involves the integration of the current with respect to voltage and has been successfully used to extract the parameters of a diode [16, 17]. The use of integration, instead of the more commonly employed differentiation, makes this method more immune to measurement errors because of the low-pass filter nature of integration. Furthermore, in the case of negligible parallel resistance ($G_{p1} \approx G_{p2} \approx 0$) and for values of $I >> I_o$, it was proved that the following function eliminates the effects of the series resistance [18]:

$$G(I,V) \equiv \frac{I\,V - 2\int_0^V I\ dV}{I} \approx n\,V_{th}\left[\ln\left(\frac{I}{I_O}\right) - 2\right] \quad . \quad (4)$$

Auxiliary function *G* is numerically calculated from the experimental data using this definition. The right hand side of (4) indicates that a plot of function *G* versus ln(*I*), for $I >> I_o$, should produce a straight line. Therefore, the slope and the intercept on the voltage axis allow the determination of the intrinsic *n* and $I_o$ parameters, respectively, without any interference from the parasitic series resistance, $R_s$. Once these two parameters are known, the value of $R_s$ can be found readily. It should be mentioned that the lower limit of integration in (4) does not have to be zero, that is, function *G* can be applied to any particular region of the forward *I-V* characteristics. This would be useful whenever it is expected that the intrinsic parameters are not constant throughout the whole characteristics and a multiple exponential model is called for.

The numerator of equation (4) represents what is known as the Integral Difference Function *D*(*I*,*V*) defined as [18]:

$$\begin{aligned} D(I,V) &\equiv \int_0^I V\ dI - \int_0^V I\ dV \equiv I\,V - 2\int_0^V I\ dV \\ &\equiv\ 2\int_0^I V\ dI\ - I\,V \end{aligned} \quad . \quad (5)$$

As before, it should be stressed that, although the integrals' lower limits shown in (5) are defined here at the origin for simplicity, this does not need be so, that is, function *D* can be applied to any particular region between any two points of the forward *I-V* characteristics. Function *D* also finds useful applications in other areas, such as for quickly calculating device harmonic distortion [19].

Substitution of (3) into (5) and integrating with respect to *I* results in a long expression in terms of Lambert *W* functions, *V*, and *I* (See Appendix A). Rewriting the terms that contain Lambert *W* functions of *V*, using equation (3), and after some algebraic manipulation, the final expression of *D*(*I*,*V*) results in a purely algebraic equation of the form:

$$D(I,V) \equiv \mathrm{D}_{\mathrm{V1}}\ V + \mathrm{D}_{\mathrm{I1}}\ I + \mathrm{D}_{\mathrm{I1V1}}\ V\ I + \mathrm{D}_{\mathrm{V2}}\ V^2 + \mathrm{D}_{\mathrm{I2}}\ I^2 , \quad (6)$$

where the five coefficients are given by:

$$\mathrm{D}_{\mathrm{I1}} = -2\ R_S\ I_0 - 2\ n\ V_T\ (1 + G_{P1} R_S\ ) \quad , \quad (7)$$

$$\begin{aligned} \mathrm{D}_{\mathrm{V1}} &= 2 R_S n V_T G_{P1} G_{P2} + 2 I_0 (R_S G_{P2} + 1) \\ &\quad + 2 n V_T (G_{P1} + G_{P2}) \end{aligned} \quad , \quad (8)$$

$$\mathrm{D}_{\mathrm{I1V1}} = 1 + 2\ R_S\ (\ G_{P1} + G_{P2}\ ) + 2\ R_S^{\ 2}\ G_{P1}\ G_{P2} \quad , \quad (9)$$

$$\mathrm{D}_{\mathrm{I2}} = -R_S\ (\ 1 + \ G_{P1}\ R_S\ ) \quad , \quad (10)$$

$$\mathrm{D}_{\mathrm{V2}} = -G_{P2} - G_{P1} - 2R_S G_{P1} G_{P2} - R_S G_{P2}^2 - \mathrm{R}_S^2 G_{P1} G_{p2}^2 \qquad (11)$$

It should be remarked that algebraic expression (6) was arrived at without using any approximations and that it is valid for any region of operation, including $I < I_o$. The intermediate steps involving the integration of $V$ with respect to $I$ in terms of Lambert $W$ functions, $V$, and $I$ are presented in Appendix A.

A close look at the five coefficients (7)-(11) indicates that they are not all independent. For example, using (9)-(11), we easily see that:

$$\mathrm{D}_{\mathrm{I1V1}}^2 = 1 + 4\,\mathrm{D}_{\mathrm{I2}}\,\mathrm{D}_{\mathrm{V2}} \quad . \qquad (12)$$

This means that there are really four independent coefficients. Therefore, for a general case of significant parasitic series resistance and both shunt conductances, the five diode model parameters, $n$, $I_o$, $R_s$ , $G_{p1}$ , and $G_{p2}$, may not be extracted uniquely. The general solution of $n$, $I_o$, $G_{p1}$, and $G_{p2}$, in terms of $R_s$, and coefficients (7) to (11) is presented in Appendix B. Nevertheless, parasitics in practical diodes are frequently modeled by a series resistance in combination with a single shunt conductance. In such cases all four model parameters may be extracted uniquely. Let us next analyze these cases.

### 3.1 Junction with shunt loss and series resistance

When the significant shunt losses occur only at the junction, and a series parasitic resistance is also present, letting $G_{P2} = 0$ , the describing equation reduces to

$$I = I_O\left( \exp\left( \frac{V - I\,R_s}{n\,V_{th}} \right) - 1 \right) + (V - I\,R_s)\,G_{p1} \quad . (13)$$

Explicit solutions for the current and voltage in terms of Lambert $W$ functions are:

$$I = \frac{nV_T}{R_S}\mathrm{W}\left\{ \frac{d_1 I_0 R_S}{nV_T} \exp\left[ \frac{d_1 (V + I_0 R_S)}{nV_T} \right] \right\} + d_1 (VG_{P1} - I_0) \qquad (14)$$

and

$$V = -nV_T\,\mathrm{W}\left\{ \frac{I_0}{nV_T G_{P1}} \exp\left[ \frac{I + I_0}{nV_T G_{P1}} \right] \right\} + \quad I\left[ R_S + \frac{1}{G_{P1}} \right] + \frac{I_0}{G_{P1}} \qquad (15)$$

In this case the coefficients that define $D(I,V)$ in (6) are:

$$\mathrm{D}_{\mathrm{I1}} = -2\,R_S\,I_0 - 2\,n\,V_T\,(1 + G_{P1} R_S) \qquad (16)$$

$$\mathrm{D}_{\mathrm{V1}} = 2I_0 + 2nV_T G_{P1} \qquad (17)$$

$$\mathrm{D}_{\mathrm{I1V1}} = 1 + 2\,R_S\,G_{P1} \qquad (18)$$

$$\mathrm{D}_{\mathrm{I2}} = -R_S\,(1 + G_{P1}\,R_S) \qquad (18)$$

$$\mathrm{D}_{\mathrm{V2}} = -G_{P1} \qquad (20)$$

Fitting (6) to the experimental data produces the coefficients given by the above equations; and from them, the four diode model parameters, $n$, $I_o$, $R_s$, and $G_{p1}$, can be determined uniquely.

### 3.2 Junction with peripheral shunt loss and series resistance

When only shunt losses at the periphery are relevant, letting $G_{P1} = 0$, the describing equation becomes

$$I = I_O\left\{ \exp\left[ \frac{V\,(1 + R_s\,G_{p2}) - I\,R_s}{n\,V_{th}} \right] - 1 \right\} + V\,G_{p2} \quad . (21)$$

Again using the Lambert $W$ function, the explicit solutions for the current and voltage are:

$$I = \frac{nV_T}{R_S}\mathrm{W}\left\{ \frac{I_0 R_S}{nV_T} \exp\left[ \frac{(V + I_0 R_S)}{nV_T} \right] \right\} - I_0 + V\,G_{P2} \qquad (22)$$

and

$$V = -\,d_2\,n\,V_T\,\mathrm{W}\left\{ \frac{I_0}{d_2 nV_T G_{P2}} \exp\left[ \frac{I + \frac{I_0}{d_2}}{nV_T G_{P2}} \right] \right\} + \frac{I + I_0}{G_{P2}} \qquad (23)$$

Now the coefficients that define $D(I,V)$ in (6) are:

$$\mathrm{D}_{\mathrm{I1}} = -2\,R_S\,I_0 - 2\,n\,V_T \qquad (24)$$

$$\mathrm{D}_{\mathrm{V1}} = +2I_0 (R_S G_{P2} + 1) + 2nV_T G_{P2} \qquad (25)$$

$$\mathrm{D}_{\mathrm{I1V1}} = 1 + 2\,R_S\,G_{P2} \qquad (26)$$

$$\mathrm{D}_{\mathrm{I2}} = -R_S \qquad (27)$$

$$\mathrm{D}_{\mathrm{V2}} = -G_{P2}\,(1 + R_S\,G_{P2}) \qquad (28)$$

As before, fitting (6) to the experimental data produces the coefficients given by equations (24) to (28). The four diode model parameters, $n$, $I_o$, $R_s$, and $G_{p2}$, can be determined uniquely from those coefficients.

### 3.3 Junction with series resistance and without shunt loss

In the not infrequent event that all shunt loss mechanisms could be neglected, letting $G_{P1} = G_{P2} = 0$ yields the case already studied by Banwell and Jayakumar [20],

$$I = I_0\left[ \exp\left( \frac{V - IR_S}{nV_T} \right) - 1 \right] \quad . \qquad (29)$$

The current can be solved explicitly (Equation. (4) in Ref. [20]) as

$$I = \frac{nV_T}{R_S} \mathrm{W}\left\{ \frac{I_0 R_S}{nV_T} \exp\left[ \frac{(V + I_0 R_S)}{nV_T} \right] \right\} - I_0 \quad . \tag{30}$$

Obviously the solution for the voltage can be easily obtained from (29) in terms of common elementary functions:

$$V = IR_S + nV_T \ln\left( \frac{I + I_0}{I_0} \right) \quad . \tag{31}$$

In this simple case the coefficients that define $D(I,V)$ in (6) are:

$$\mathrm{D}_{\mathrm{I1}} = \text{-}2\, R_S\, I_0 \text{ - } 2\, n\, V_T \tag{32}$$

$$\mathrm{D}_{\mathrm{V1}} = 2I_0 \tag{33}$$

$$\mathrm{D}_{\mathrm{I1V1}} = 1 \tag{34}$$

$$\mathrm{D}_{\mathrm{I2}} = \text{-}\, R_S \tag{35}$$

$$\mathrm{D}_{\mathrm{V2}} = 0 \tag{36}$$

Again, fitting (6) to the experimental data produces the coefficients defined by the above equations. Notice that two of the diode model parameters $I_o$ and $R_s$, are given directly by two of the coefficients, (33) and (35), respectively. Then, the third parameter, $n$, is determined from (32).

### 3.4 Junction with shunt loss and without series resistance ( $R_S = 0$ )

Conversely, if the series resistance is negligible, letting $R_S \rightarrow 0$ and denoting $G_P = G_{P1} + G_{P2}$ the combination of any shunt loses present, yields directly the explicit solution for the current in terms of elementary functions,

$$I = I_0 \left[ \exp\left( \frac{V}{nV_T} \right) - 1 \right] + G_P\, V \quad . \tag{37}$$

However solving for the voltage involves a transcendental equation whose explicit solution again may be obtained in terms of Lambert $W$ functions:

$$V = -nV_T \mathrm{W}\left[ \frac{I_0}{nV_T G_P} \exp\left( \frac{I + I_0}{nV_T G_P} \right) \right] + \frac{I + I_0}{G_P} \quad . \tag{38}$$

In this case the coefficients that define $D(I,V)$ in (6) are:

$$\mathrm{D}_{\mathrm{I1}} = \text{-}\, 2\, n\, V_T \tag{39}$$

$$\mathrm{D}_{\mathrm{V1}} = +2I_0 + 2nV_T G_P \tag{40}$$

$$\mathrm{D}_{\mathrm{I1V1}} = 1 \tag{41}$$

$$\mathrm{D}_{\mathrm{I2}} = 0 \tag{42}$$

$$\mathrm{D}_{\mathrm{V2}} = \text{-}\, G_P \tag{43}$$

Once again, two of the diode model parameters $n$, and $G_p$, are given directly by two of the coefficients, (39) and (43), respectively. The third parameter, $I_o$, is finally determined from (40).

## 4. Discussions

Figure 2 presents synthetic *I-V* characteristics obtained from (2) using parameters values of $I_o = 10^{-12}$A, $n = 1.5$, $G_{p1} = 0$ and various combinations of $R_s$ and $G_{p2}$. ¨The calculations were done using Maple [21] with 20 digits precision. The ideal case of $R_s = 0$ and $G_{p2} = 0$, shown by large hollow squares, is a straight line. The case when $R_s$ = 1kΩ is significant and $G_{p2} = 0$, indicated by small solid squares, produces a straight line for low voltage that bends down for high voltage (i.e. the effects $R_s$ become important at high voltage). The case when only $G_{p2}$ is significant ($G_{p2} = 1\mu$S and $R_s = 0$), is presented by small solid circles. It is a straight line at high voltage and bends up at low voltage (i.e. the effects $G_{p2}$ are important at low voltage).

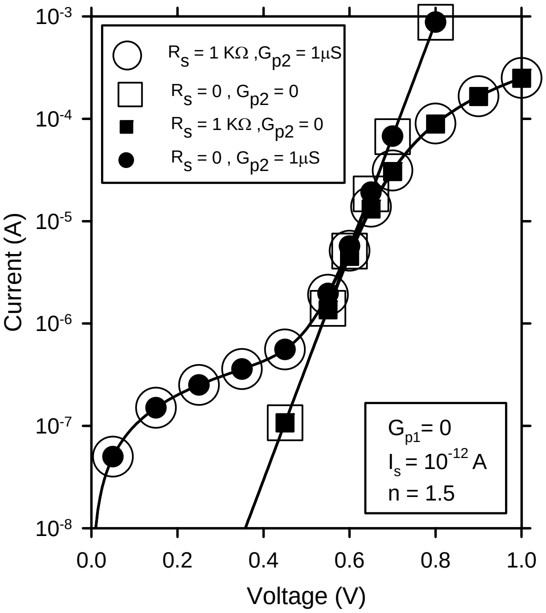

Fig. 2 Test synthetic *I-V* characteristics simulated using parameters values of $I_o = 10^{-12}$A, $n = 1.5$ , and various combinations of $R_s$ and $G_{P2}$.

When $R_s$ and $G_{p2}$ are both simultaneously significant ($R_s$ = 1kΩ and $G_{p2} = 1\mu$S) is presented by large hollow circles. As it is evident from the graph, the plot in this case does not present any linear segment because the effects $R_s$ and $G_{p2}$ overlap. Therefore the intrinsic parameters may not be obtained in the traditional way directly from any portion of the *I-V* characteristics.

Figure 3 presents the calculated $D$ as a function of terminal voltage for the $I$-$V$ characteristics illustrated in the previous figure. We observe in this figure that the ideal case ($R_s$ = 0 and $G_{p2}$ = 0) and the case of only significant $G_{p2}$ ($G_{p2}$ = 1µS and $R_s$ = 0) both display the same $D(V)$ everywhere. This is because the operator $D$ when represented as a function of of the terminal voltage $V$ eliminates the effects of $G_{p2}$. We also observe that in the case of only significant $R_s$ ($R_s$ = 1kΩ and $G_{p2}$ = 0), $D$ differs from the ideal case only at high voltage where $R_s$ becomes important. The case of $R_s$ = 1kΩ and $G_{p2}$ = 1µS produces the same $D$ for all $V$ as the case $R_s$ = 1kΩ and $G_{p2}$ = 0.

Figure 4 shows the calculated $D$ as a function of terminal current for the $I$-$V$ characteristics illustrated in figure 2. The figure indicates that the ideal case ($R_s$ = 0 and $G_{p2}$ = 0) and the case of significant $R_s$ ($R_s$ = 1kΩ and $G_{p2}$ = 0) both display the same $D(I)$ everywhere. This is because the operator $D$ when represented as a function of $I$ eliminates the effect of $R_s$. We also observe that for the case of only significant $G_{p2}$ ($R_s$ = 0 and $G_{p2}$ = 1µS), $D$ differs from the ideal case only at low voltage where the effect of $G_{p2}$ is significant. The case when $R_s$ = 1kΩ and $G_{p2}$ = 1µS gives the same $D$ for all $V$ as the case $R_s$ = 0 and $G_{p2}$ = 1µS. The previous combinations, as well as several other additional cases, were simulated and the quadratic equation of $D$ as a function of current and voltage, defined in (6), was then used to extract the simulated parameters. In all cases the extraction procedure succeeded in producing the exact original parameters, within computational accuracy.

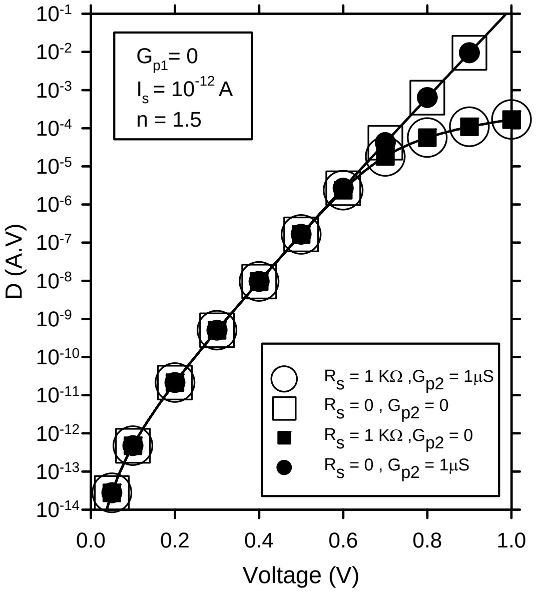

Fig. 3 Calculated D as a function of voltage.

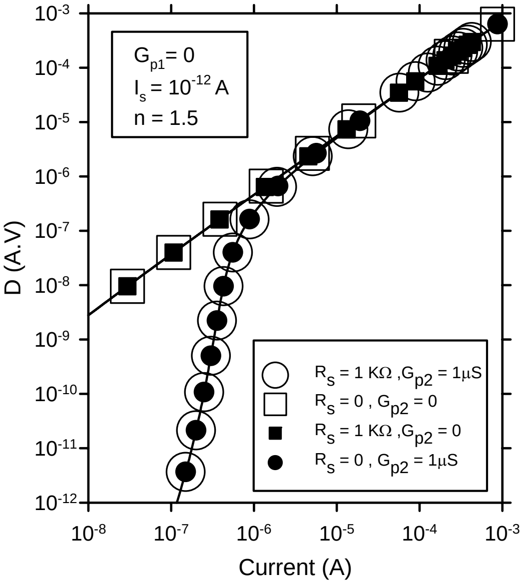

Fig. 4 Calculated D as a function of current.

## 5. Conclusions

A new method to extract the intrinsic and extrinsic model parameters of semiconductor junctions containing parasitic resistance and shunt conductance has been presented. The Integral Difference Function $D$ was applied to the exact explicit analytical solutions of the junction's $I$-$V$ characteristics. Since the resulting equation of $D$ is expressed in terms of Lambert $W$ functions, parameter extraction by direct numerical fitting of the equations was not attempted as it would result computationally impractical. Instead, the resulting $D$ is reduced to a purely algebraic equation with terms in $I$, $V$, $IV$, $I^2$, and $V^2$. The intrinsic and extrinsic model parameters can then be readily determined from the coefficients of these terms. The applicability of the procedure was exemplified for the cases of only series resistance, only shunt conductance, and two series resistance and shunt conductance combinations. The quadratic two-dimensional fitting process represents a fast and accurate parameter extraction procedure. The application of the method to non-ideal synthetic $I$-$V$ characteristics demonstrates that it is theoretically exact, its correctness depending only on numerical computation inaccuracies.

## Appendix A

The integration of $V$, defined in (3), with respect to $I$ in terms of Lambert $W$ functions is:

$$\int_0^I V dI = P_1 - P_2 \ \mathrm{W}[x\exp(y)] - \frac{P_2}{2}\mathrm{W}[x\exp(y)]^2 \quad , \quad \text{(A1)}$$

where

$$P_1 \equiv nV_T I_0 \quad + \frac{\left(1+R_S G_{P2}\right) I_0{}^2 + I\left(I + IR_S G_{P1} + 2I_0\right)}{2\left(G_{P1}+G_{P2}+G_{P1}G_{P2}\ R_S\right)} \text{ , (A2)}$$

$$P_2 \equiv \frac{G_{P1}+G_{P2}+G_{P1}G_{P2}R_S}{\left(1+R_S\ G_{P2}\right)}\left(nV_T\right)^2 \quad , \qquad \text{(A3)}$$

$$y \equiv \frac{I + I_0\left(1+G_{P2}R_S\right)}{nV_T\left(G_{P1}+G_{P2}+G_{P1}G_{P2}R_S\right)} \qquad \text{(A4)}$$

and

$$x \equiv \frac{I_0\left(1+G_{P2}R_S\right)}{nV_T\left(G_{P1}+G_{P2}+G_{P1}G_{P2}R_S\right)} \qquad \text{(A5)}$$

The variables $P_1$ and $P_2$ have units of power and *x* and *y* are dimensionless.

## Appendix B

The general solution of *n*, *I<sub>o</sub>*, *G<sub>p1</sub>*, and *G<sub>p2</sub>*, in terms of $R_S$, $\mathrm{D}_{\mathrm{V1}}$, $\mathrm{D}_{\mathrm{I1}}$, $\mathrm{D}_{\mathrm{I1V1}}$, $\mathrm{D}_{\mathrm{V2}}$ and $\mathrm{D}_{\mathrm{I2}}$ is as follows. Solving for $G_{P1}$ in (10),

$$G_{P1} = -\frac{\mathrm{D}_{\mathrm{I2}} + R_S}{R_S^2} \quad , \qquad \text{(B1)}$$

and solving for $G_{P2}$ in quadratic equation (11),

$$G_{P2} = \frac{-1-2R_S G_{P1} + \left(1-4R_S\mathrm{D}_{\mathrm{V2}} - 4\mathrm{R}_{\mathrm{S}}^2 G_{P1}\mathrm{D}_{\mathrm{V2}}\right)^{1/2}}{2R_S\left(1+R_S G_{P1}\right)} \quad . \text{ (B2)}$$

Next, $I_0$ and *n* are obtained by solving the system of two linear equations described by (7) and (8),

$$I_0 = \frac{1}{2}\left(\mathrm{D}_{\mathrm{V1}} + G_{P1}\mathrm{D}_{\mathrm{I1}} + G_{P2}\mathrm{D}_{\mathrm{I1}} + G_{P1}R_S\mathrm{D}_{\mathrm{V1}} + G_{P1}R_S G_{P2}\mathrm{D}_{\mathrm{I1}}\right) \text{ , (B3)}$$

and

$$n = -\frac{R_S\mathrm{D}_{\mathrm{V1}} + R_S G_{P2}\mathrm{D}_{\mathrm{I1}} + \mathrm{D}_{\mathrm{I1}}}{2\,V_T} \quad . \qquad \text{(B4)}$$

It is important to point out that given a set of values for $\mathrm{D}_{\mathrm{V1}}$, $\mathrm{D}_{\mathrm{I1}}$, $\mathrm{D}_{\mathrm{V2}}$ and $\mathrm{D}_{\mathrm{I2}}$ ($\mathrm{D}_{\mathrm{I1V1}}$ is obtained from(12) ) defines a unique current voltage characteristic which can be generated with numerous combinations of $R_S$ *n*, $I_o$, $G_{p1}$, and $G_{p2}$. For example, given a set of values for $\mathrm{D}_{\mathrm{V1}}$, $\mathrm{D}_{\mathrm{I1}}$, $\mathrm{D}_{\mathrm{V2}}$, $\mathrm{D}_{\mathrm{I2}}$, the values of *n*, $I_o$, $G_{p1}$, and $G_{p2}$ can be obtained from (B1-B4) for any arbitrary value of $R_S$.

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